Covariant formulation of pion-nucleon scattering *

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Abstract

A covariant model of elastic pion-nucleon scattering based on the Bethe-Salpeter equation is presented. The kernel consists of s- and u-channel N and $\Delta(1232)$ poles, along with ρ and σ exchange in the t-channel. A good fit is obtained to the s- and p-wave phase shifts up to the two-pion production threshold.

Pion-nucleon (πN) scattering is an important example of a strong interaction, and as such plays a significant role in many other nuclear reactions involving pions, for example pion photoproduction. Ideally a theory describing πN scattering should be derived from Quantum Chromodynamics (QCD), but since QCD is not at present solvable for low energies, it is necessary to use a chirally invariant effective Lagrangian, where the degrees of freedom are baryons and mesons, rather than quarks and gluons. Following the success of meson-exchange models in describing the nucleon-nucleon interaction, a number of meson-exchange models for πN scattering have been developed over the last few years [1]. These models invariably begin with an effective Lagrangian which describes the couplings between the various mesons and baryons. The tree-level diagrams obtained from this Lagrangian are then unitarized in a 3-dimensional approximation to the Bethe-Salpeter (BS) equation [2]. Convergence is guaranteed by the introduction of phenomenological form factors at each vertex. There are an infinite number of 3-dimensional reductions to the BS equation, and there is no overwhelming reason to choose one particular approximation over any other. Here we describe a covariant model of elastic πN scattering in which the BS equation is solved without any reduction to 3-dimensions.

In principle, the exact $\pi N \leftarrow \pi N$ amplitude for a given Lagrangian can be obtained from the BS equation, with a potential consisting of all 1- and 2-particle irreducible diagrams, and dressed propagators in the πN intermediate state. Since it is impossible to construct such a potential, as it would contain an infinite number of diagrams, it is common practice to truncate this kernel and include only the tree-level diagrams. Furthermore, if only two-body unitarity is required to be maintained, then the dressed nucleon propagator is replaced by a bare propagator with a pole at the physical mass.

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Our kernel consists of s- and u-channel N and $\Delta(1232)$ exchanges, along with ρ and σ exchange in the t-channel. We do not include any higher baryon resonances because these contributions are not expected to be significant for πN scattering below the two-pion production threshold. The couplings to the pion field are always through derivative couplings, as required by chiral symmetry.

There is an ambiguity as to the choice of propagator for a particle with spin-3/2. The most commonly used propagator is the Rarita-Schwinger propagator, which is known to have both spin-3/2 as well as background spin-1/2 components [3]. Other forms have been introduced by Williams [4] and Pascalutsa [5], which each have only a spin-3/2 component. In the present paper we use the Rarita-Schwinger propagator.

To guarantee the convergence of all integrals, we need to associate with each vertex a cut-off function. We take this cut-off function to be the product of form factors that depend on the 4-momentum squared of each particle present at the vertex [1]. Each form factor is chosen to be of the form

$$f(q^2) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2} \right),$$

where q^2 is the 4-momentum squared of the particle and m is the mass. A different cutoff mass Λ is used for each particle.

The s-channel pole terms present in the potential become dressed when the BS equation is iterated. Therefore bare coupling constants and masses are used in the N and Δ s-channel pole diagrams in the potential. The bare nucleon parameters are determined by requiring that in the P_{11} partial wave, there is a pole at the physical nucleon mass with a residue related to the physical πNN coupling constant. Since the dressed Δ has a width, it would be necessary to analytically continue the BS equation into the complex s-plane in order to carry out the renormalization for the Δ. Rather than doing this we treat the bare Δ mass and the bare $\pi N\Delta$ coupling constant as free parameters. Since the P_{33} partial wave is dominated by the s-channel Δ pole diagram, the bare Δ parameters are essentially fixed by the P_{33} phase shifts. While only the s-channel diagrams in the potential are dressed by the ladder BS equation, Pearce and Afnan [6] have shown that if 3-body unitarity is satisfied, then the u-channel diagrams also become dressed. We approximate this by using physical masses and coupling constants in the u-channel N and Δ diagrams.

We solve the BS equation by first expanding the nucleon propagator in the πN intermediate state into positive and negative energy components, and then sandwiching the resulting equation between Dirac spinors. This gives two coupled 4-dimensional integral equations which are reduced to 2-dimensional integral equations after a partial wave decomposition. A Wick rotation [7] is carried out in order to obtain equations suitable for numerical solution. With our choice of form factors, there is no interference from the form factors when carrying out the Wick rotation, provided the cutoff masses are large enough. The cutoff masses also have to be chosen large enough so that unphysical thresholds, which are generated by form factor singularities and propagator singularities pinching the integration contour, are far above the two-pion production threshold. More details are given in [8].

The renormalized πNN coupling constant is fixed at its physical value of $g^2_{\pi NN}/4\pi = 13.5$. The nucleon renormalization procedure fixes the bare N parameters. The remaining parameters are determined in a fit to the phase shifts and scattering lengths from the SM95

partial wave analysis of Arndt et al [9]. For the cutoff masses we obtain $\Lambda_N = 3.12$, $\Lambda_\pi = 1.73$, $\Lambda_\rho = 4.67$, $\Lambda_\Delta = 4.86$, and $\Lambda_\sigma = 1.4$ (all in GeV). The coupling constants are $g_{\rho\pi\pi}g_{\rho NN}/4\pi = 3.2$, $\kappa_\rho = 2.57$, $g_{\sigma\pi\pi}g_{\sigma NN}/4\pi = -0.3$, $f^2_{\pi N\Delta}/4\pi = 0.44$, and $f^{(0)2}_{\pi N\Delta}/4\pi = 0.35$. The remaining masses are $m^{(0)}_\Delta = 2.13$ GeV and $m_\sigma = 700$ MeV. With these parameters, the renormalization procedure gives $m^{(0)}_N = 1.37$ GeV and $g^{(0)2}_{\pi NN}/4\pi = 3.8$.

We obtain a good fit to the s- and p-wave phase shifts. The resulting phase-shifts are shown in Figure 1, and the scattering lengths and volumes are shown in Table I. In general our coupling constants are consistent with those used in the other πN models. Assuming universality ($g_\rho \equiv g_{\rho\pi\pi} = g_{\rho NN}$), our value of $g^2_\rho/4\pi$ is close to that obtained from the decay $\rho \to 2\pi$, i.e. $g^2_\rho/4\pi = 2.8$, and κ_ρ is close to the value $\kappa_\rho = 3.7$ arising from vector meson dominance. Our physical $\pi N\Delta$ coupling constant is slightly larger than the value $f^2_{\pi N\Delta}/4\pi = 0.36$ obtained from calculations of the width of the Δ (the use of the larger coupling was necessary in order to obtain a good fit to the P_{13} and P_{31} phase shifts). The contribution of σ-exchange is very small, while the u-channel Δ diagram provides a large amount of attraction in all partial waves except S_{31} and P_{33}. The attraction in the P_{11} partial wave is dominated by ρ-exchange and the u-channel Δ. Notice that some additional attraction is required for high energies in the S_{11} partial wave.

The cutoff masses turn out to be quite large, with the result that the dressing is significant, as is evident from the large size of the bare N and Δ masses. The baryon self energies are dominated by the one-pion loop diagrams. In view of the significance of the dressing, it is interesting to examine the effect of the dressing on the πNN form factor. We can calculate a renormalized cutoff mass by comparing the dressed πNN vertex, with the nucleons on-mass-shell and the pion off-shell, to a monopole form factor. We find $\Lambda^R_\pi = 1.17$ GeV (recall that the bare cutoff mass is 1.73 GeV). Therefore the vertex dressing softens the πNN form factor.

REFERENCES

[1] B.C. Pearce and B.K. Jennings: Nucl. Phys. **A528**, 655 (1991); F. Gross and Y. Surya: Phys. Rev. **C47**, 703 (1993); C. Schütz et al.: Phys. Rev. **C49**, 2671 (1994); C.T. Hung, S.N. Yang, T.-S.H. Lee: J. Phys. **G20**, 1531 (1994); V. Pascalutsa and J.A. Tjon: Nucl. Phys. **A631**, 534c (1998).

[2] E.E. Salpeter and H.A. Bethe: Phys. Rev. **84**, 1232 (1951).

[3] M. Benmerrouche, R.M. Davidson, N.C. Mukhopadhyay: Phys. Rev. **C39**, 2339 (1989).

[4] H.T. Williams: Phys. Rev. **C31**, 2297 (1985).

[5] V. Pascalutsa: hep-ph/9802288.

[6] B.C. Pearce and I.R. Afnan: Phys. Rev. **C34**, 991 (1986).

[7] G.C. Wick: Phys. Rev. **96**, 1124 (1954).

[8] A.D. Lahiff and I.R. Afnan: in preparation.

[9] R.A. Arndt et al.: Phys. Rev. **C52**, 2120 (1995)

TABLES

	BSE	SM95		BSE	SM95
S_{11}	0.184	0.175	S_{31}	-0.101	-0.087
P_{11}	-0.079	-0.068	P_{31}	-0.045	-0.039
P_{13}	-0.037	-0.022	P_{33}	0.181	0.209

TABLE I. Scattering lengths and volumes obtained from the Bethe-Salpeter equation (BSE). Units are $m_\pi^{-(2\ell+1)}$.

FIGURES

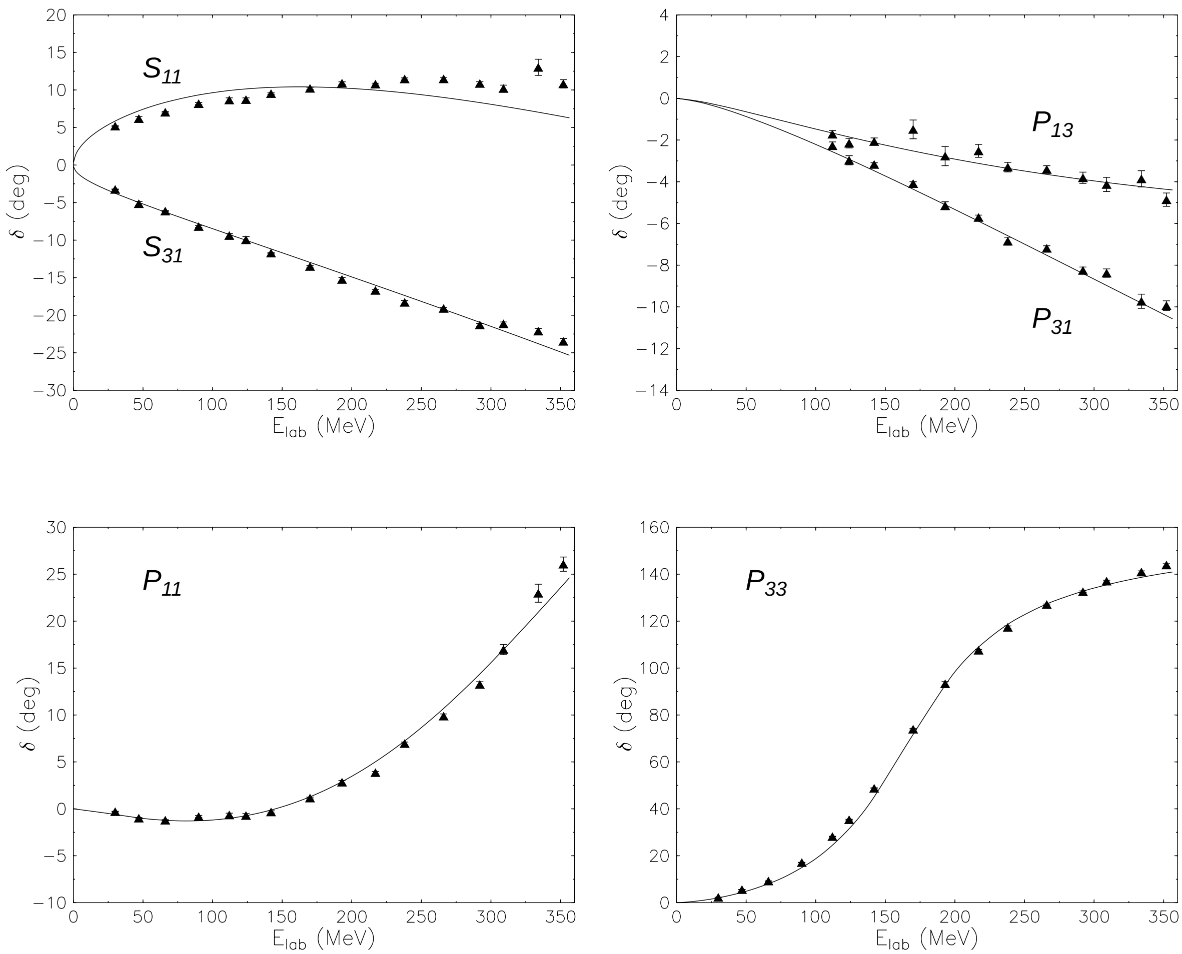

FIG. 1. The phase shifts obtained from the Bethe-Salpeter equation are shown versus the pion laboratory energy, compared to the VPI SM95 partial wave analysis.